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Technical Note

Buoyant convection in a side-heated cavity under gravity and oscillations

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1. Introduction

Buoyant convection in a side-heated square cavity of a Boussinesq fluid has served as a benchmark configuration [1]. The two vertical sidewalls are maintained at different constant temperatures T_h and T_c, respectively, and the horizontal walls are insulated (see Fig. 1). Concern is with the case of large Rayleigh number Ra ($\equiv g_0 \alpha \Delta T L^3/\nu\kappa$) where, g_0 denotes gravity; α, the coefficient of thermometric expansion; ΔT, the horizontal temperature difference ($\equiv T_h - T_c$); L, the height of the square cavity; ν, the kinematic viscosity; and κ, the thermal diffusivity of the fluid.

As observed in [2–4], buoyant convection in an enclosure with time-periodic boundary conditions has emerged to be a topic of increasing interest of late. One fundamental issue is the existence of resonance, which was first pointed out by Lage and Bejan [5] and Lage et al. [6–9]. They considered the case when the heat flux at one vertical sidewall varies with time in a square wave. Resonance is manifested by the intensification of convective activities in the interior core. It was shown that, at the proper resonance frequency, the amplitude of Nusselt number fluctuation in the central region of the cavity is maximized. [10–12] asserted that resonance is expected to occur when the time-periodic external condition excites the eigenfrequencies of the system. For a differentially heated cavity, in which the temperature at one vertical wall varies periodically, [10–12] demonstrated that the system eigenfrequencies are characterized by the modes of internal gravity oscillations, which are supported by the prevailing stratification.

The present account probes into the principal features of resonance in a side-heated square cavity under a time-periodic gravity vector. It is recalled that, in the buoyant flow models of [5,10], the imposed time-periodicity was given to the boundary condition at the vertical sidewall. In contrast, in the present set-up, the entire apparatus is subject to a time-varying gravitational environment. Examples can be found in the thermo-fluid systems under severe mechanical vibrations. The problem is of immediate concern to the designers of space vehicles, which operate in micro-gravity with g-jitters.

Fu and Shieh [13] performed a numerical study of thermal convection, which was driven simultaneously by gravity and vertical vibration of the container. A relatively small Rayleigh number ($Ra = 10^4$) was considered, and the changes in flow patterns and heat transport rates with varying vibration frequencies were illustrated. The formulation of [13] was such that the vertical vibrational acceleration was $-A\omega^2 \sin \omega t$; therefore, the amplitude of acceleration ($-A\omega^2$) was a function of the frequency ω. This implies that the sep-

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arate effects of amplitude and of vibration frequency cannot be readily delineated.

The aim of the present brief communication is to provide an additional description of the basic mechanism of resonance in seemingly different buoyant systems at large *Ra*. The common thread is the ascertainment that resonance takes place when the frequency of external fluctuations matches the eigenfrequencies of the system, which are identified to be the modes of internal gravity oscillations. In this note, independent effects of amplitudes and frequency of oscillations on the resonance characteristics are studied.

2. Model

The vertically downward constant gravity is g_0, and two cases are considered for the fluctuating parts of acceleration: (case 1), in the vertical direction $g_y = \varepsilon_y g_0 \sin(ft)$; (case 2), in the horizontal direction $g_x = \varepsilon_x g_0 \sin(ft)$.

The governing time-dependent Navier–Stokes equations, in properly nondimensionalized form, read

$$\frac{\partial U}{\partial \tau} + \frac{\partial}{\partial X}(U^2) + \frac{\partial}{\partial Y}(UV) = -\frac{\partial P}{\partial X} - \varepsilon_x \sin(\omega\tau)\theta + \left(\frac{Pr}{Ra}\right)^{1/2} \nabla^2 U, \tag{1}$$

$$\frac{\partial V}{\partial \tau} + \frac{\partial}{\partial X}(UV) + \frac{\partial}{\partial Y}(V^2) = -\frac{\partial P}{\partial Y} + (1 + \varepsilon_y \sin(\omega\tau))\theta + \left(\frac{Pr}{Ra}\right)^{1/2} \nabla^2 V, \tag{2}$$

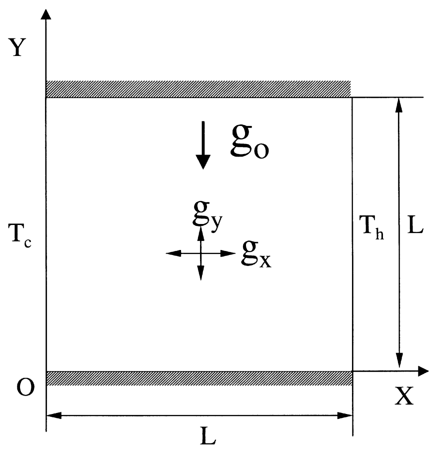

Fig. 1. Schematic diagram of flow configuration.

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial}{\partial X}(U\theta) + \frac{\partial}{\partial Y}(V\theta) = \left(\frac{1}{PrRa}\right)^{1/2} \nabla^2 \theta, \tag{3}$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{4}$$

in which nondimensional quantities are defined as

$$\tau = t(RaPr)^{1/2}\frac{\kappa}{L^2}, \qquad (U, V) = (u, v)(RaPr)^{-1/2}\frac{L}{\kappa},$$

$$(X, Y) = \frac{(x, y)}{L}, \qquad \theta = \frac{T - T_c}{T_h - T_c},$$

$$P = \frac{(p + \rho g_0 y)L^2}{\rho\kappa^2 RaPr}.$$

In the above equations, (U, V) are velocity components in the (X, Y) directions, and the Prandtl number $Pr \equiv \nu/\kappa$. Note that the time is scaled by using the reciprocal of the Brunt–Väsällä frequency, N based on the horizontal temperature contrast, i.e. $N \equiv (RaPr)^{1/2}\kappa/L$. The appropriateness of this scaling can be justified from the fact that N characterizes the stratification of basic-state flow (see [10]).

The associated boundary conditions are

$$U = V = \frac{\partial \theta}{\partial Y} = 0, \quad \text{at } Y = 0, 1;$$

$$U = V = \theta = 0, \quad \text{at } X = 0;$$

$$U = V = 0, \qquad \theta = 1, \quad \text{at } X = 1.$$

In Eqs. (1)–(4), case 1 refers to $\varepsilon_x = 0$; and for case 2, $\varepsilon_y = 0$. Also, note that the nondimensional fluctuating frequency $\omega \equiv f/N$.

The above problem was solved numerically by utilizing the well-established SIMPLER algorithm [14]. A (61 × 61) mesh network was deployed with the implementation of grid stretchings. The time step was typically $\Delta\tau = 2\pi/(1024\omega)$. The numerical methodologies and computational details have been amply documented, and the sensitivity and convergence tests were conducted in sufficient detail. As the computational procedures, no new claims are made here; the calculations were carried out almost in a routine manner. Comparisons of the present results with the available data were made, and the present results were found to be highly consistent with the previous data.

3. Results and discussion

The solutions of the basic steady-state ($\varepsilon_y = \varepsilon_x = 0$) are denoted by subscript ss. For convenience, a physical variable ϕ, the departure from the basic-state is expressed by $G(\phi)$, i.e.

$$G(\phi) = \frac{\phi - \phi_{ss}}{\phi_{ss}},$$

and the amplitude of fluctuation of ϕ is shown by $A(\phi)$, i.e.

$$A(\phi) = \frac{\mathrm{Max}[\phi(\tau)] - \mathrm{Min}[\phi(\tau)]}{2}, \qquad \tau_0 \leq \tau \leq \tau_0 + \frac{2\pi}{\omega}.$$

Also, at the vertical line $X = X$, the Nusselt number is defined as

$$Nu(\tau)_{X=X} = \int_0^1 \left[U\theta(RaPr)^{1/2} - \frac{\partial\theta}{\partial X} \right]_{X=X} \mathrm{d}Y.$$

A comprehensive series of numerical solutions were acquired for $Ra = 10^7$ and $Pr = 0.7$. The amplitude of the external excitation was set to be small, ε_x or $\varepsilon_y \leq 0.03$. A compilation of the computed results is displayed in Fig. 2 in the form of $A(Nu)/\varepsilon$ at the centerline $X = 0.5$ versus ω. Obviously, this plot exhibits the normalized amplitude of fluctuating heat transport in the interior, $A(Nu)/\varepsilon$, for a given external excitation frequency ω. It is evident that $A(Nu)$ peaks at a particular frequency $\omega_r \cong 0.66$ for both the cases 1 and 2. This is indicative of resonance, as observed by [5,10] for the cases of time-periodic thermal boundary condition imposed at the vertical sidewall. The present results are in close agreement with these preceding observations both in the value of ω_r and in the general shape of $A(Nu)/\varepsilon$–ω curve. Furthermore, for the present range of small-amplitude external excitations, ε_x or $\varepsilon_y \ll 1$, $A(Nu)/\varepsilon$–ω curves are almost independent of ε_x or ε_y. This suggests that, in the linear range ε_x or $\varepsilon_y \ll 1$, $A(Nu)$ is proportional to the strength of excitation.

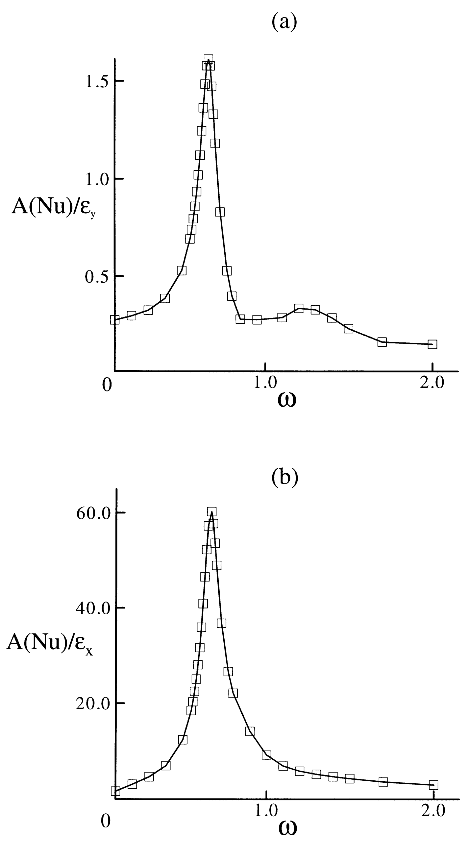

Fig. 2. Plots of $A(Nu)/\varepsilon$ vs. ω, (a) case 1, $\varepsilon_x = 0$, $\varepsilon_y = 0.01$; (b) case 2, $\varepsilon_x = 0.01$, $\varepsilon_y = 0$.

In an effort to explain the mechanism of resonance, [12] argued that the modes of internal gravity oscillation in the interior characterize the eigenfrequencies of the present buoyant flow system. In accordance with the developments of Paolucci and Chenoweth [15], the fundamental mode of these oscillations is given as $\omega_i = S/\sqrt{2}$. In the above equation, S indicates the overall vertical stratification in the interior, i.e. $S \cong \partial\theta/\partial y$. A curve-fitting was made to the computed temperature field of the basic-state flow, and these exercises yield $\omega_i \cong 0.68$, which is in satisfactory agreement with the computed resonance frequency $\omega_r \cong 0.66$. These again reinforce the prior ascertainment that resonance occurs when the modes of internal gravity oscillations are excited.

In Fig. 2, it is worth pointing out that the numerical value of $A(Nu)/\varepsilon$ for case 2 is much larger than for case 1. This implies that the magnification of amplitude of heat transfer is far more effective when the oscillation is made in the horizontal, than in the vertical, direction. As emphasized in [10], when the system is in resonance, the interior isotherms undergo a periodic tilting. The horizontal oscillation therefore is more effective in producing the tilting of isotherms, and the numerical results are consistent with this line of physical reasoning.

Time histories of flow and thermal fields under resonance condition over a cycle are exemplified in Fig. 3. For the case of vertical oscillation (see Fig. 3A), the changes in effective gravity are quantitative in nature. Consequently, when ε_y is very small, the resulting variations in the flow patterns are not prominent. However, in the case of horizontal oscillation (see Fig. 3B), the direction of effective gravity deviates from the vertical direction, and the resulting effect on the interior is more direct and pronounced. It is notable in Fig. 3B that the general directions of isotherms in the cavity are tilting sideways over a cycle, which causes qualitative changes in the global flow pattern.

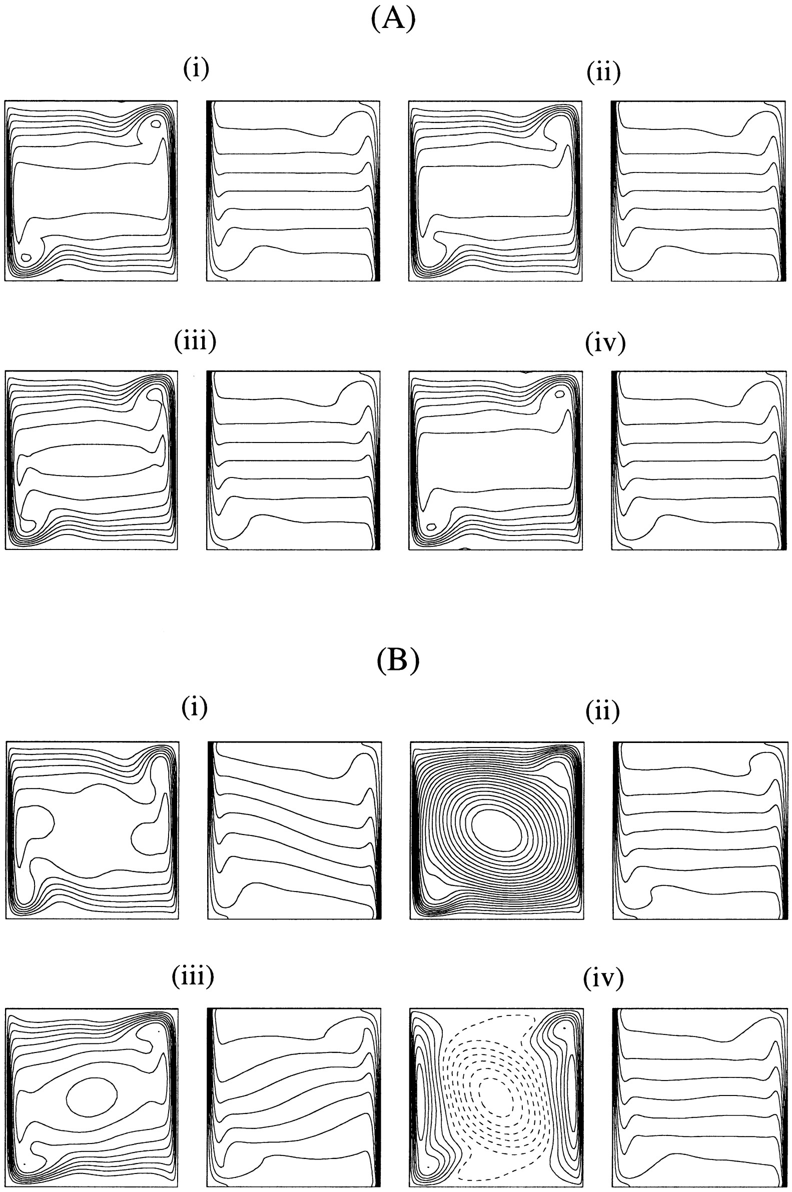

Fig. 3. Flow pattern (in the left box) and temperature fields (in the right box) are shown in each pair of the boxes, (A) case 1, $\varepsilon_x = 0$, $\varepsilon_y = 0.03$, $\omega = 0.66$; (B) case 2, $\varepsilon_x = 0.03$, $\varepsilon_y = 0$, $\omega = 0.66$. Time instants are (i) the start of the cycle, (ii) 1/4 cycle, (iii) 1/2 cycle and (iv) 3/4 cycle.

4. Concluding remarks

The computed results suggest that resonance takes place when the frequency of oscillations matches the basic mode of internal gravity oscillations. At the resonance frequency, $A(Nu)$ is maximized, which suggests intensification of convective activities in the interior. The oscillation in the vertical direction brings forth mostly quantitative changes, but the changes are far more pronounced and qualitative when the oscillation is in the horizontal direction.

Acknowledgements

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